





# Constraint-Based Reasoning in Static Analysis and Testing

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# Test Data Generation & Static Analysis Based on Symbolic Execution & Constraint Solving

# Testing & Static Analysis

## Testing:

- Test data/case generation/preparation \*\*\*
- Test case execution
- Test result analysis / fault localization ...

Static Analysis (of source code) \*\*\*

# Path-oriented/sensitive Analysis

\* program path → path condition (PC)



**symbolic execution**

(or backward substitution)

\* The path is feasible (or executable) iff the PC is satisfiable.

**Constraint Solving**

# Symbolic Execution

giving *symbolic* values as input to the program,  
and simulating the program's behaviour

- ◆ [Boyer et al. 1975] [King 1976] [Clarke 1976]
- ◆ Many research groups working on this technique ...
- ◆ **Path feasibility analysis and constraint solving**  
[Zhang 2000] [Zhang-Wang 2001]
- ◆ **Constraint Solving and Symbolic Execution**  
[Zhang VSTTE2005]

# A path in bubble-sort

- $i = n-1;$
- $@ i > 0;$
- $indx = 0;$
- $j = 0;$
- $@ j < i;$
- $@ a[j+1] < a[j];$
- $temp = a[j];$
- $a[j] = a[j+1];$
- $a[j+1] = temp;$
- $indx = j; \quad j = j+1;$
- $@ j \geq i;$
- $i = indx;$
- $@ i \leq 0;$

## ■ Path condition:

$n-1 > 0$

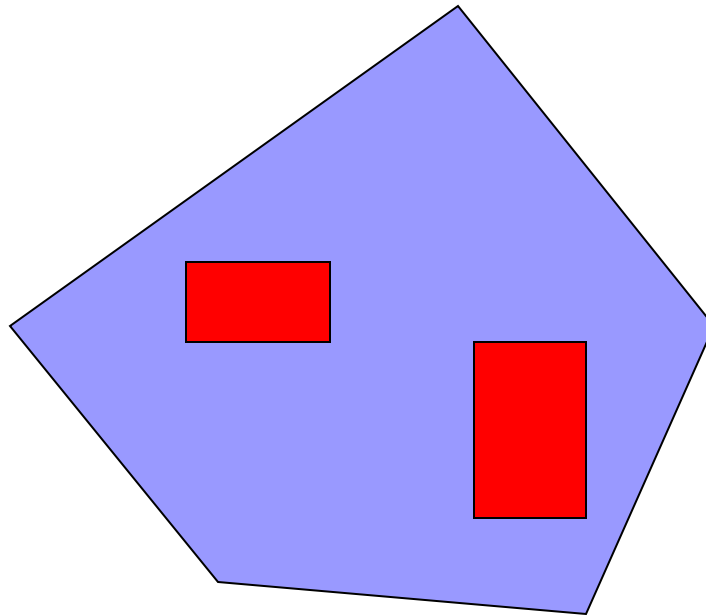
$a[1] < a[0]$

$n-1 \leq 1$

Input data:  $n = 2$

$a[]: \{ 3, 2 \}$  or  $\{ 8, 1 \}$  or ...

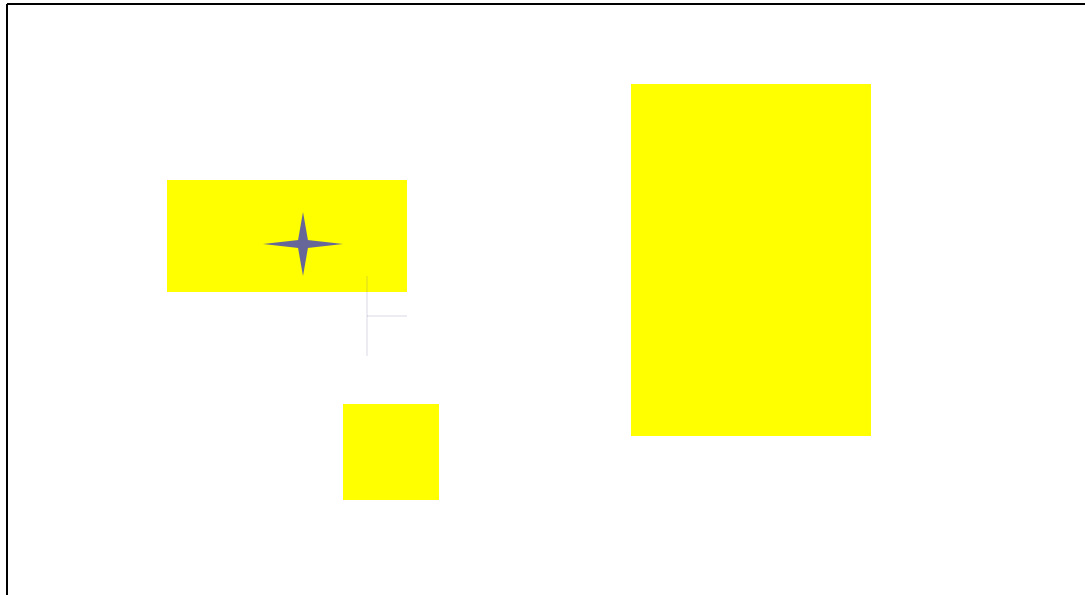
# Symbolic Execution



Checking **areas** (rather than **points**) in the input space.

# Test Generation and Bug Finding via Symbolic Execution

- Finding one point in any area





# Satisfiability Checking (SAT solving)

- Example Input:

p cnf 3 3

1 2 3 0

-1 -2 0

-2 -3 0

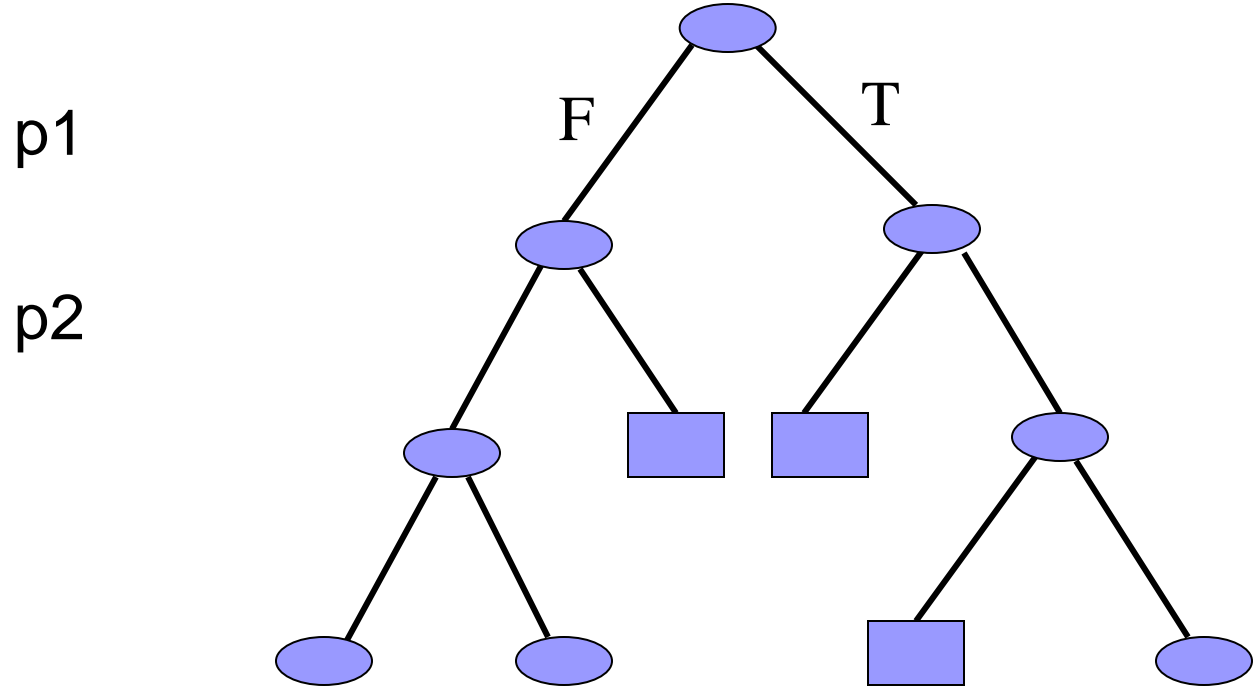
denoting the following formula:

$(p_1 \text{ OR } p_2 \text{ OR } p_3) \text{ AND}$

$(\text{NOT } p_1 \text{ OR } \text{NOT } p_2) \text{ AND}$

$(\text{NOT } p_2 \text{ OR } \text{NOT } p_3)$

# Binary search tree



# Satisfiability Modulo Theories (SMT) solvers – CVC3/CVC4, Yices, Z3, ...

- $x_3, x_2, x_1, x_0$ : INT;

CHECKSAT ( $x_0 \geq 0$  AND  $x_0 \leq 9$  AND  
 $x_1 \geq 0$  AND  $x_1 \leq 9$  AND  $x_2 \geq 0$  AND  
 $x_2 \leq 9$  AND  $x_3 \geq 0$  AND  $x_3 \leq 9$  AND  
( $x_0 > 0$  OR  $x_1 > 0$  OR  $x_2 > 0$  OR  $x_3 > 0$ )  
AND  $1000 * x_0 + 100 * x_3 + 10 * x_2 + x_1 =$   
 $2000 * x_3 + 200 * x_2 + 20 * x_1 + 2 * x_0$ );

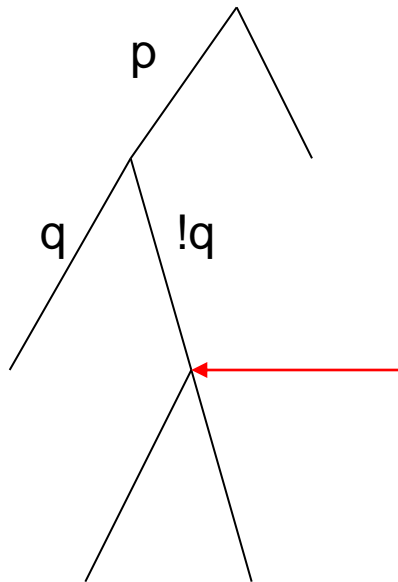
# Constraint Solver: BoNuS (1999-2001)

[Zhang 2000] [ZhangWang 2001]

## ■ Example.

```
enum { Male, Female } gender;  
int age;  
bool b = (age > 18);  
bool married;  
{  
    and(not(b), married);  
}
```

# Search proc. (simple SMT solving)



$p: (x > 3)$

$q: (2 * x > 5)$

Check the feasibility of:  
 $x > 3; 2 * x \leq 5$

Linear programming:  
`lp_solve`

# Constraint Solving + Symbolic Execution

## [Zhang VSTTE 2005 (LNCS 4171)]

- verify – or find bugs in – certain programs
- check the error messages produced by other static analyzers, to eliminate some false alarms
- automate an important part of unit testing, i.e., generating test cases (input data) for the program
- generate test cases for black-box testing or model-based testing, if a proper specification (like EFSM) is provided.

# Unit Testing – Stmt/branch coverage

Examples: GNU coreutils [XZ 2006]

- **remove\_suffix()** in `basename.c`
- **cat()** in `cat.c`
- **cut\_bytes()** in `cut.c`
- **parse\_line()** in `dircolors.c`
- **set\_prefix()** in `fmt.c`
- **attach()** in `ls.c`
- **bsd\_split\_3()** and **hex\_digit()** in `md5sum.c`

# Example. GNU make: dir.c

```
char *dosify(char filename[20]) {  
    ...  
    for (i = 0; *filename != '\0' && i < 8 &&...; i++) {  
        *df = *filename; df++; filename++;  
    }  
    if (*filename != '\0') {  
        *df = *filename; df++; filename++;  
        for (i = 0; *filename != '\0' && i < 3 &&...; i++) {  
            *df = *filename; df++; filename++;  
        }  
    }  
}
```

**Test suite:** 3 test cases → 100% branch coverage



# Static Analysis of C programs

- inter-procedural, path sensitive tools for finding *memory leak*
  - [Xu-Zhang, 2008] on top of LLVM
  - [Xu-Zhang-Xu, 2011] **melton**, on top of Clang static analyzer
- Canalyze -- a tool for finding various kinds of bugs (e.g., NULL pointer dereferencing; undefined return value; ...)

# Bugs Found in Open Source Software

Software	KLoC	Undef. value	NULL ptr	Mem. leak	Use after free
libosip2-4.0.0	28.9		✓	✓	✓
libosip2-3.6.0	29.0			✓	
lighttpd-1.4.32	46.3	✓		✓	✓
Openssh-5.9p1	89.8			✓	
wget-1.13	91.8	✓		✓	
sqlite-3.7.11	139.2			✓	
Coreutils-8.15	202.3		✓		
Coreutils-8.17	211.9				✓
sed-4.2	30.4	✓			
glibc-2.15	1020.5		✓		

# Error in Openssh-5.9p1

```
//in file sshconnect2.c
authmethod_get(...) {
    ...
    for( ; ;) {
        if ((name = match_list(...)) == NULL){//Allocate heap space to name
        }
        ...
        if (...) {
            ...
            xfree(name);
            return ...;
        }
    }
}
if (name) xfree(name);
}
```

# Ex. bug report -- bftpd

From: "Jesse Smith" <jessefrgsmith@yahoo.ca>  
Date: 2013-5-28  
Subject: Re: Some potential bugs in bftpd-3.8  
To: "Zhenbo Xu" <zhenbo1987@gmail.com>

I had a chance to examine your bug reports for Bftpd. **All of the problems you reported are correct. The memory handling for bftpd\_cwd\_mappath() was an especially bad bug.**

All of these bugs have been fixed in my copy of the code and I will be releasing a new version of Bftpd soon ...

# Finding Bugs Related to Floating-point numbers

- Divide by zero
- One operand much larger than the other
- ...

To detect such problems, we may need to solve constraints like:

$$((11 * a2 * b2 - b6 - 121 * b4) * 16777216) < 3$$

# Finding witnesses for data race bugs

Data race – severe concurrency bug

- [Said et al. 2011]
- ...
- [Huang-Meredith-Rosu 2014]

We need to solve constraints like:

$X_{10} - X_3 = 1$ ;  $X_5 < X_7$  or  $X_9 < X_2$ ; ...

# Detecting Resource Leak in Android Apps

## Resources:

- exclusive (e.g., Camera)
- memory consuming (e.g., MediaPlayer)
- energy consuming (e.g., SensorManager)

## Resource request and release operations

Tool: **Relda** [Guo et al. 2013]

Benchmarks: Baidu, Taobao, Tencent, ...

# Example.

```
private void initCamera() throws IOException
{ if(!blfPreview)
    { //If the camera is not in preview mode, turn it on.
      mCamera = Camera.open(); }
  if (mCamera != null && !blfPreview)
  { ..... mCamera.startPreview(); blfPreview = true; }
}

private void resetCamera()
{ if (mCamera != null && blfPreview)
  { mCamera.stopPreview();
    mCamera.release();
    ..... }
}
```



# Technology Transfer

- need from industry
- power of the tool



➤ Commercialization ... ??

# Combinatorial Testing

- Black-box testing technique
- The system-under-test (SUT) has a set of parameters/components, each of which can take some values.
- Example:
  - ✓ Browser: IE, Chrome, Firefox, ...
  - ✓ Operating system: Linux, Windows XP, ...
  - ✓ Manufacturer: Dell, Lenovo, HP, ...

# Constraints in CT

- Example of constraints:

`not ((Browser==IE) && (OS==Linux))`

- [Arcuri and Briand 2012] "in the presence of constraints, random testing can be worse than combinatorial testing"

# Constraints used everywhere

- static program analysis (bug finding)
- combinatorial testing
  
- [DeMillo-Offutt ~1990], ...
- KLEE, SAGE, ...
- Workshop on the Constraints in Software Testing, Verification and Analysis (CSTVA)

# All kinds of Constraints

- Linear inequalities:  $x+2y < 3$
- Integer difference constraints:  $x-y < 2$
- Non-linear constraints:  $2xy+z = 8$
- Propositional formulas:  $(p \vee \sim q) \wedge r$
- SMT formulas:  $(x-y>5) \vee (x+2y<16)$
- ...



# Extensions to SMT Solving

# Extension to SMT solving (I)

Finding 1 solution

Finding the **best** solution

Logic +  
arithmetic

SMT

**SMT-opt**

Linear  
Inequalities

LP, ...

Linear  
programming

# Optimization w.r.t. complex constraints

- Linear Programming

min.  $f(X)$

s.t.  $Ax \leq b$

- SMT optimization

min.  $f(X)$

s.t. constraints in SMT form



# A Simple Example

min.  $x - y$

subject to:

$$(((y + 3x < 3) \rightarrow (30 < y)) \vee (x \leq 60))$$

$$\wedge ((30 < y) \rightarrow \neg (x > 3) \wedge (x \leq 60))$$

# Stress Testing – test data gen.

- Extract paths from some model (e.g., activity diagram).
- From the path, obtain resource consumption information.
- Generate constraint solving/optimization problems.
- Solve them !

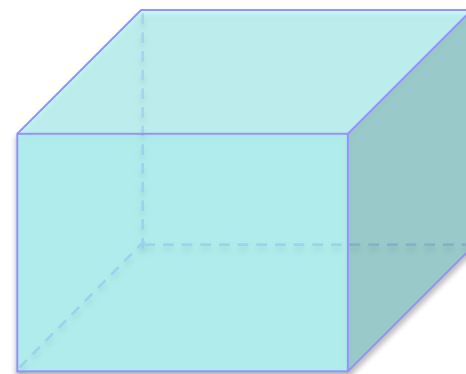
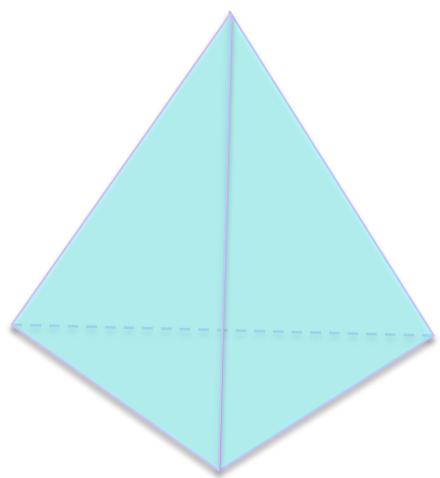
[Zhang-Cheung 2002]

min:  $-100 (x_{00}-x_{01}) -1200 (x_{10}-x_{11}) -1800 (x_{20}-x_{21}) -56 (x_{30}-x_{31}) -1500 (x_{40}-x_{41}) -150 (x_{50}-x_{51}) -1440 (x_{60}-x_{61}) -25 (x_{70}-x_{71}) -200 (x_{80}-x_{81}) -230 (x_{90}-x_{91})$

$u_1 = 1; r_1 = 1; v_0 \leq 25; s_0 \leq 25; u_0 = v_1; r_0 = s_1;$   
 $u_1 \leq u_0; x_{01} = u_1 \text{ or } x_{11} = u_1 \text{ or } x_{21} = u_1; x_{00}$   
 $\leq u_0; x_{10} \leq u_0; x_{20} \leq u_0; x_{01} = x_{21}; x_{10} = x_{20};$   
 $x_{00} - x_{01} \leq 5; x_{10} - x_{11} \leq 6; x_{20} - x_{21} \leq 15; v_1$   
 $\leq v_0; x_{31} = v_1 \text{ or } x_{41} = v_1; \dots$

# Extension to SMT solving (II)

	solution?	#of solutions
Logic+ arithmetic	SMT	Vol.comput.
prop. Logic	SAT	model counting



# Compute volume / solution density

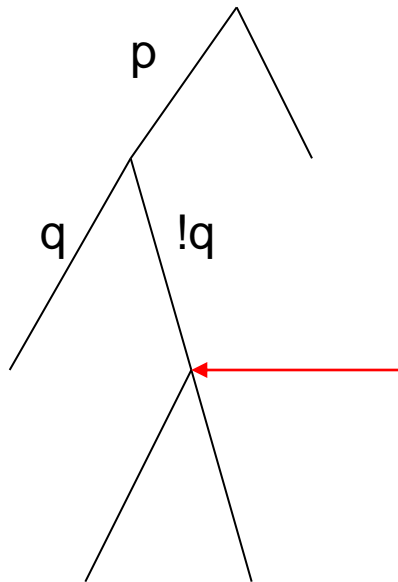
- Given an SMT formula (a set of constraints), compute the volume of its solution space (or its solution density).

■ Example.  $\Phi :=$

$$(((y+3x < 1) \rightarrow (30 < y)) \vee (x \leq 60)) \wedge ((30 < y) \rightarrow \neg(x > 3) \wedge (x \leq 60))$$

- High complexity: #p-hard even for a single convex polyhedron

# Solution counting [MaLiuZhang 2009]



$p: (x > 3)$

$q: (2 * x > 5)$

Check the #of solutions of:  
 $x > 3; 2 * x \leq 5$

Vol. computing for polytopes:  
**vinci**

# Estimate the volume of polytopes

- Simple Monte-Carlo algorithm  
[Liu-Zhang-Zhu 2007]
- *PolyVest* [Ge-Ma-Zhang 2013]





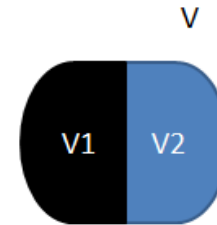
# **A Testing Problem as a By-product**

- How do we know the “reliability” of the method?
- How do we know the accuracy of the results?

# Testing PolyVest

## Relation r1:

$$V = V1 + V2$$



## Results:

$$Deviation(V, V') = \frac{V' - V}{V}$$

Dimension	Vol(V)	Vol(V1)	Vol(V2)	Vol(V1+V2)	Deviation
5	814.03	573.792	254.765	828.557	+1.78%
8	829.167	323.116	435.406	758.522	-8.52%
14	16961.6	8594.56	8302.33	16896.89	-0.38%
20	1.101e+12	2.412e+11	8.332e+11	1.074e+12	-2.45%

# Testing PolyVest

## Relation r2:

V1:  $Ax \leq b$

V2:  $Ax \leq nb$

Dimension of  $x$  is  $m$

Volume relation:  $V2 = V1 * n^m$

## Results:

$$Deviation(V, V') = \frac{V' - V}{V}$$

Dimension	Vol(V1)	n	$n^m$	expected	Vol(V2)	Deviation
5	848.758	5	3125	2.652e+06	2.620e+06	-1.21%
8	815.157	3	6561	5.348e+06	5.259e+06	-1.66%
14	15631.2	3	4.782e+06	7.485e+010	7.585e+10	+1.34%
20	1.048e+12	2	1.049e+06	1.099e+18	1.135e+18	+3.28%

# Constraint Solving and Symbolic Execution [Zhang VSTTE2005]

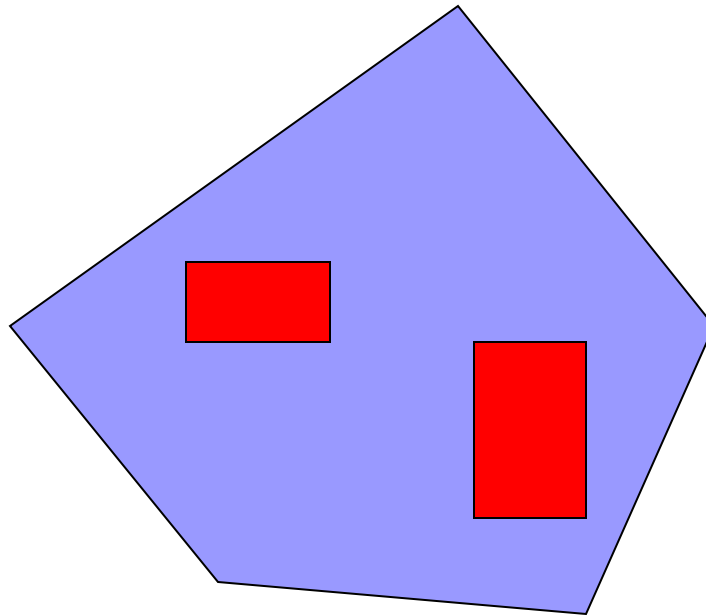
- Verification
- Static analysis
- Testing

“We can also perform **other kinds of analysis** which are not so related to the correctness of programs.” (page 544)



# Branch/Path Execution Frequency Computation

# Symbolic Execution



Checking **areas** (rather than **points**) in the input space.  
What are the sizes of the areas? How much is covered?

# Branch selection-- Example

```
int x;  
@ ((x <= 100) && (x > 20))  
{  
  x = x - 10;  
  if (x > 30)  
    ... //TRUE branch  
  else  
    ... //FALSE branch  
}
```

- TRUE branch  
75% (3/4)
- FALSE branch  
25% (1/4)

Constraints:

- $(a \leq 100) \&\& (a > 20)$   
 $(a - 10 > 30)$
- $(a \leq 100) \&\& (a > 20)$   
 $(a - 10 \leq 30)$

```
int x;
@ ((x <= 50) && (x > 20))
{
  x = x - 10;
  if (x > 30)
    ... //TRUE branch
  else
    ... //FALSE branch
}
```

- TRUE branch: 1/3
- FALSE branch: 2/3

Constraints:

- $(a \leq 50) \&\& (a > 20)$   
 $(a - 10 > 30)$
- $(a \leq 50) \&\& (a > 20)$   
 $(a - 10 \leq 30)$



# Path execution frequency -- Example.

```
int gettop(s,lim)
    char s[];  int lim;
{
    int i, c;
    while ((c=getchar()) == ' ' || c == '\t' || c == '\n') ;
    if (c != '.' && (c < '0' || c > '9')) return(c);
    s[0] = c;
    for(i = 1; (c=getchar()) >= '0' && c <= '9'; i++)
        if (i < lim) s[i] = c;
    if (c == '.') {    if (i < lim) return(c);    ...    }
}
```



- Path 1

$1 \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 27.$

$XP(\text{Path1}) \approx 0.945$

- Path 2

$1 \rightarrow 2 \rightarrow 4 \rightarrow 7 \rightarrow 11 \rightarrow$   
 $19 \rightarrow 20 \rightarrow 27.$

$XP(\text{Path2}) \approx 0.021$



# Performance Estimation Based on Symbolic Execution & Volume Computing

# Estimating A Program's Performance

- $\delta(P)$  -- the number of solutions of the path condition ( $\rightarrow$  path exec. frequency).
- Symbolic benchmarking
  - Generate some paths;
  - Calculate the performance of each path --  $PIND(P_i)$
  - Estimate Performance of the program:

$$PIND(P1) * \delta(P1) + PIND(P2) * \delta(P2) + \dots$$

# Example. Bubble sort

```
for(i = 0; i < N-1; i++)  
    for(j = N-1; j > i; j--) {  
        if (a[j-1] > a[j])  
            swap(a[j-1], a[j]);  
    }
```

# Analysis of bubble sort

N=4: 24 paths. ( $\delta$ : ~ the same for each path)

- For each path, the number of comparisons is the same (6).
- But the number of swaps is different, ranging from 0 to 6. The total number of swaps for all the 24 paths is 72.
- **On average** (when N=4), it needs 3 swaps of array elements and 6 comparisons between array elements.

# Example. FIND [Hoare 1971]

Input: array A; size N; int f;

Output:

$A[0], A[1], \dots, A[f-1]$

$\leq A[f]$

$\leq A[f+1], \dots, A[N-1].$

# benchmarking -- I

- Randomly choose some paths

(N=8, f=3)

Path	nComp	nSwap	Delta(P)
w11	9	3	16303680
w16	9	2	12191040
w18	9	3	16303680
w20	9	2	12191040



# benchmarking -- II

Randomly generate some input data

- { -2, 5, 6, 3, 1, 0, -7, 6 };
- { 2, 0, -2, -8, 4, -4, 5, 1 };

Path	nSwap	Delta(P)
R1	4	4075920
R6	6	87516

➤ Average #of swaps: 4.04

$$(4075920*4 + 87516*6) / (4075920+87516) \approx 4.04$$

# Symbolic Benchmarking

- Symb. execution  $\rightarrow$  symb. benchmarking
  - one symb. exec. == many conc. exec.
- Given the program
  - generate paths from the flow graph; or run the program a number of times
  - calculate the PIND and Delta values
  - get the estimated performance of the prog. (weighted sum of the PINDs)



# Reliability of Component-Based Software Systems

# Reliability of Component-based Syst.

- ▣ reliability of components → reliability of systems ?
  - execution frequency calculation
  - ...
- ▣ Existing Works:
  - [Hamlet-Mason-Woit 2001] Theory of software reliability based on components
  - ▣ System design – control structures: sequences, conditionals, loops, ...
  - ▣ ("A supporting tool would ...")



## □ Existing Works:

[Palviainen, Evesti, Ovaska 2011] The reliability estimation, prediction and measuring of component-based software

- Model-based reliability prediction/measuring
- State transition matrix – each element of the matrix  $P_{sa}$  denotes the probability of moving from state  $s$  to state  $a$ .
- A tool chain ...

# Summary

- All kinds of constraints
  - Linear/non-linear inequalities
  - SAT ----  $(p \text{ or not } q) \text{ and } (\text{not } p \text{ or } q)$
  - SMT ----  $(x+y \geq 3) \text{ and good and } (x-y \neq 8)$
- Decision problem → counting, optimization
- Testing, analysis, reliability, security ...



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